Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level In Pure Mathematics (WMA11) Paper 01

Question	Scheme	Marks
1	$\int\left(10x^4-\frac{3}{2x^3}-7\right)\mathrm{d}x=2x^5+\frac{3}{4}x^{-2}-7x+c$	M1A1A1
		(3)
		Total 3

Notes:

M1: For increasing one of the powers of x by 1 e.g. $x^4\to ...x^5$ or $x^{-3}\to ...x^{-2}$ or $7\to ...7x$

A1: Any **two** correctly integrated terms, unsimplified or simplified.

Allow e.g. $\frac{10}{4+1}x^{4+1},\ -\frac{3}{-4}x^{-3+1},\ -7x^1$

A1: $2x^5+\frac{3}{4}x^{-2}-7x+c$ all correct and simplified in one expression and including "+ c".

Allow correct simplified equivalents for $\frac{3}{4}x^{-2}$ e.g. $0.75x^{-2}$, $\frac{3}{4x^2}$ but do **not** allow $-7x^1$ for $-7x$ or $\frac{2}{1}$ for 2

Award this mark once a correct simplified expression is seen and apply isw if necessary.

Condone poor notation e.g. spurious integral signs, "dx", $\frac{\mathrm{d}y}{\mathrm{d}x}=...$ and look for the correct expression.

Question	Scheme	Marks
2(i)(a)	$2^{n+3} = 2^n \times 2^3 = 8m$	B1
		(1)
(b)	$16^{3n} = \left(2^4\right)^{3n}$	M1
	$= 2^{12n} = \left(2^n\right)^{12} = m^{12}$	A1
		(2)

Notes:

(i)(a)

B1: For 8*m* (Condone 8M). Do **not** allow $2^3 m$.

(b)

M1: For writing 16^{3n} correctly as an expression involving a power of 2.

Examples: $\left(2^4\right)^{3n}$, $\left(2^2\right)^{6n}$, $\left(2^{3n}\right)^4$, $\left(2^n\right)^{12}$, $\left(2^{12}\right)^n$, 2^{12n}

A1: For m^{12} (Condone M^{12})

Correct answer only scores both marks.

(ii)	$x\sqrt{3}-3=x+\sqrt{3} \Rightarrow x\sqrt{3}-x=3+\sqrt{3} \Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3} \Rightarrow x=...$	M1
	$=\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\pm\left(\sqrt{3}+1\right)}{\pm\left(\sqrt{3}+1\right)}$	M1
	$=\dfrac{\pm\left(4\sqrt{3}+6\right)}{\pm\left(3-1\right)}=3+2\sqrt{3}$	A1
		(3)

Notes:

In this part, the A1 depends on both M marks

(ii)

M1: Collects *x* terms to one side, factorises and makes *x* the subject

It is for processing the given equation and reaching $x=\dfrac{\alpha+\beta\sqrt{3}}{\gamma+\delta\sqrt{3}}$, $\alpha,\beta,\gamma,\delta\neq 0$

M1: Correct attempt to rationalise their denominator.

Not formally dependent but requires $\dfrac{...}{\gamma+\delta\sqrt{3}}=\dfrac{...}{\gamma+\delta\sqrt{3}}\times\dfrac{\gamma-\delta\sqrt{3}}{\gamma-\delta\sqrt{3}}$ or equivalent.

Sight of e.g. $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$ is sufficient for this mark **but may be implied** by e.g.

$\dfrac{3+\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{3-1}$

A1: For $3+2\sqrt{3}$ with at least one intermediate step e.g.

$\dfrac{3+\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{3-1}$ or $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{4\sqrt{3}+6}{2}$ or $\dfrac{3+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{3\sqrt{3}+3+3+\sqrt{3}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$

Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.

Example of insufficient work:

$$x\sqrt{3}-3=x+\sqrt{3} \Rightarrow x\sqrt{3}-x=3+\sqrt{3} \Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3} \Rightarrow x=\frac{3+\sqrt{3}}{\sqrt{3}-1}$$

$$=\frac{3+\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=3+2\sqrt{3}$$

Scores M1M1A0

Note that:

$$x\sqrt{3}-3=x+\sqrt{3} \Rightarrow x\sqrt{3}-x=3+\sqrt{3} \Rightarrow x\left(\sqrt{3}-1\right)=3+\sqrt{3} \Rightarrow x=\frac{3+\sqrt{3}}{\sqrt{3}-1}=3+2\sqrt{3}$$

Scores M1M0A0

Alternatives for part (ii)

(ii) ALT 1	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(p+q\sqrt{3}\right)\sqrt{3}-3=p+q\sqrt{3}+\sqrt{3}\Rightarrow p\sqrt{3}+3q-3=p+\sqrt{3}+q\sqrt{3}$ $\Rightarrow p=q+1,\ 3q-3=p$	M1
	$\Rightarrow p=q+1,\ 3q-3=p\Rightarrow p=...,q=...$	M1
	$x=3+2\sqrt{3}$	A1

Notes:

M1: Substitutes $x=p+q\sqrt{3}$, collects terms, compares coefficients and forms 2 equations in p and q
It is for obtaining 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

M1: Solves simultaneously with evidence of algebra i.e. answers not just written down from a calculator.
Not formally dependent but requires the solution of 2 equations of the form $\alpha p+\beta q=\gamma,\ \alpha,\beta,\gamma\neq 0$

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.

(ii) ALT 2	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow 3x^2-6\sqrt{3}x+9=x^2+2\sqrt{3}x+3$ $\Rightarrow 2x^2-8\sqrt{3}x+6=0$	M1
	$x^2-4\sqrt{3}x+3=0\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{\left(4\sqrt{3}\right)^2-4\times1\times3}}{2}$	M1
	$\Rightarrow x=\dfrac{4\sqrt{3}\pm\sqrt{36}}{2}\Rightarrow x=2\sqrt{3}\pm3$ $x=2\sqrt{3}+3$	A1

Notes:

M1: Squares both sides. Must obtain a 3 term quadratic expression on both sides and collect terms to one side.

M1: Solves a 3TQ of the form $px^2+q\sqrt{3}x+r=0$ by a correct non-calculator method.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

(ii) ALT 3	$x\sqrt{3}-3=x+\sqrt{3}\Rightarrow\left(x\sqrt{3}-3\right)\left(x-\sqrt{3}\right)=\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ $\Rightarrow\sqrt{3}x^2-6x+3\sqrt{3}=x^2-3\Rightarrow\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0$	M1
	$\left(\sqrt{3}-1\right)x^2-6x+3\sqrt{3}+3=0\Rightarrow x=\dfrac{6\pm\sqrt{36-4\left(\sqrt{3}-1\right)\left(3\sqrt{3}+3\right)}}{2\left(\sqrt{3}-1\right)}$ $=\dfrac{6\pm\sqrt{36-24}}{2\left(\sqrt{3}-1\right)}=\dfrac{6\pm\sqrt{12}}{2\left(\sqrt{3}-1\right)}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}=\dfrac{3\pm\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$	M1
	$=\dfrac{3\sqrt{3}+3\pm3\pm\sqrt{3}}{2}=2\sqrt{3}+3,\ \sqrt{3}$ $x=2\sqrt{3}+3$	A1

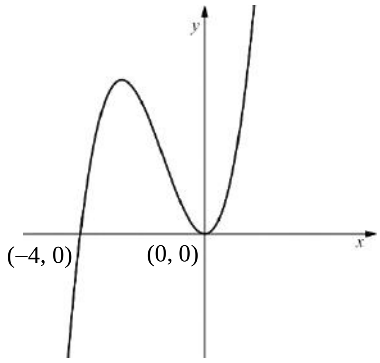
Notes:

M1: Multiples both sides by e.g. $\left(x-\sqrt{3}\right)$ and collects terms to one side.
Must obtain a 3 term quadratic expression on lhs and a 2 or 3 term quadratic expression on rhs.

M1: Solves a 3TQ of the form $\left(A+B\sqrt{3}\right)x^2+Cx+D+E\sqrt{3}=0,\ A,B,C,D,E\neq 0$ by a correct non-calculator method **and** then attempts to rationalise the denominator as in the main scheme.

A1: For $3+2\sqrt{3}$. Allow $2\sqrt{3}+3$. It is for this expression and not for just p = 3, q = 2 unless $3+2\sqrt{3}$ is seen.
If any other answers are offered and clearly not rejected score A0.

		Total 6

Question	Scheme	Marks
3(a)		B1B1B1
		(3)

Notes:

If there is any ambiguity regarding coordinates, the sketch takes precedence.

B1: Same shape translated parallel to the x-axis with no evidence **from the x-intercepts** (if labelled) that the transformation is anything other than a translation. So the x intercepts (if labelled) must be 4 units apart. Ignore any coordinates on or near the maximum.

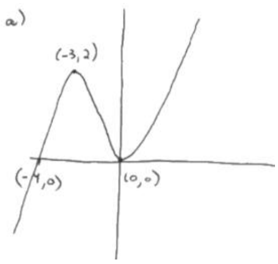
B1: Minimum at the origin. The origin does not have to be labelled.

B1: Passes through (−4, 0) or at least "reaches" this point from above or below.

Allow just −4 marked in the correct place and condone (0, −4) as long as it is marked in the correct place.

Allow to score away from the sketch as long as their curve passes through (or reaches) this point but must be identified correctly as (−4, 0) and correspond to the sketch.

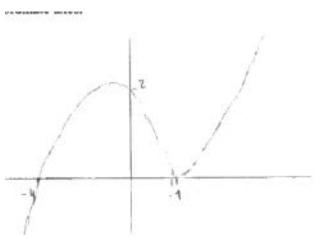
See examples below:



B1: Translation (the maximum at (−3, 2) can be ignored)

B1: Minimum at the origin

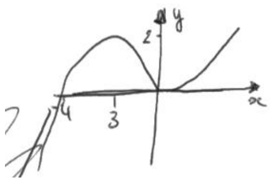
B1: Passes through (−4, 0)



B0: The x intercepts are contradictory for a translation (not 4 units apart)

B0: No minimum at the origin

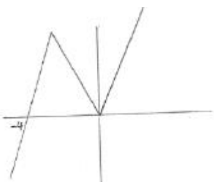
B1: Passes through (−4, 0)



B1: Translation

B1: Minimum at the origin (the intention is clear)

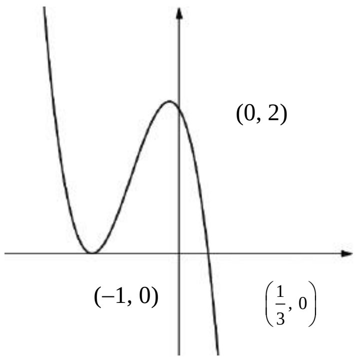
B1: Passes through (−4, 0)



B0: Incorrect shape

B1: Minimum at the origin

B1: Passes through (−4, 0)

(b)		B1B1B1
		(3)
		Total 6

Notes:

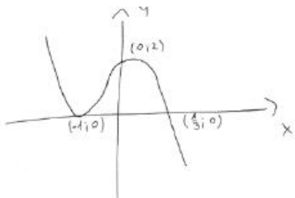
If there is any ambiguity regarding coordinates, the sketch takes precedence.

B1: Reflection in the y-axis. This requires a minimum on the negative x-axis and a maximum to the left of the y-axis in quadrant 2. If the maximum is on the y-axis score B0.
The curve must continue upwards to the left of the y-axis to reach a maximum in quadrant 2.
The curve must extend into quadrant 4 as shown.

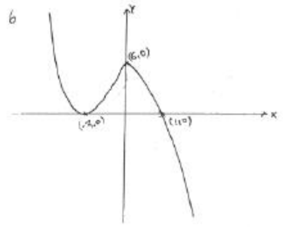
B1: Minimum at (−1, 0). Allow just −1 marked in the correct place and condone (0, −1) as long as it is marked in the correct place. Allow to score away from the sketch but must be identified correctly as (−1, 0) and correspond to the sketch.

B1: Passes through (0, 2) and passes through or touches $\left(\frac{1}{3}, 0\right)$. For both points, allow just the intercepts marked in the correct place and condone (2, 0) and/or $\left(0, \frac{1}{3}\right)$ as long as they are marked in the correct place. Allow to score away from the sketch but must be identified correctly as (0, 2) and $\left(\frac{1}{3}, 0\right)$ and correspond to the sketch.

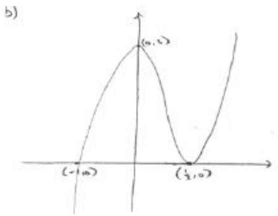
See examples below:



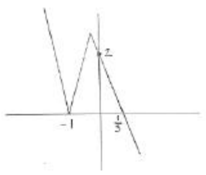
B0: Maximum in quadrant 2
B1: Minimum at (−1, 0)
B1: Passes through (0, 2) and $\left(\frac{1}{3}, 0\right)$



B0: Maximum on the y-axis (we would have tolerated the slight "pointy" maximum)
B0: Minimum at (−2, 0)
B0: Does not pass through (0, 2) and $\left(\frac{1}{3}, 0\right)$



B0: Not a reflection in the y-axis
B0: No minimum at (−1, 0)
B1: Passes through (0, 2) and touches $\left(\frac{1}{3}, 0\right)$



B0: Must be a curve – a reflection in the y-axis with the same shape.
B1: Minimum at (−1, 0)
B1: Passes through (0, 2) and $\left(\frac{1}{3}, 0\right)$

Question	Scheme	Marks
4(a)	$x^2+kx-9=-3x^2-5x+k \Rightarrow 4x^2+kx+5x-9-k\,(=0)$	M1
	$b^2-4ac=0 \Rightarrow (k+5)^2-4\times4(-9-k)=0$	M1
	$k^2+26k+169=0$*	A1*
		(3)
(b)	$k^2+26k+169=0 \Rightarrow k=-13$	B1
	$k=-13 \Rightarrow 4x^2-8x+4=0 \Rightarrow x=...$	M1
	$(1,\ -21)$	A1
		(3)
		Total 6

Notes:

Mark (a) and (b) together

(a)

M1: Equates both curves and collects terms to one side. Allow slips but there must be some attempt to collect terms to one side. The " = 0" may be implied by their attempt at the discriminant.

M1: Attempts to find the discriminant "b^2-4ac" of a 3TQ. May be seen as an attempt at e.g. $b^2-4ac=0$ or equivalent e.g. $b^2=4ac$ or may be seen embedded in an attempt at the quadratic formula.
It requires b and c both of the form $pk+q,\quad p,q\neq0$ and requires a as a constant.
If clearly a wrong formula/expression is used e.g. "b^2+4ac" this scores M0

A1*: Achieves the printed answer with no errors and sufficient working shown.
The " = 0" must appear at least once before the printed answer unless they start with $b^2=4ac$.

(b)

B1: Correct value of k. Condone $x = -13$ if it is subsequently used as a value for k.

M1: Either:

- substitutes their value of k into their equation obtained by equating the curves and collecting terms from part (a) and solves a 3TQ for x by any correct method including a calculator **or**
- substitutes their k into the 2 given equations, equates, collects terms to one side and solves a 3TQ for x by any correct method including a calculator

A1: Correct coordinates. Allow as e.g. $x = 1,\quad y = -21$

Correct answer only in (b) scores 3/3

Question	Scheme	Marks
5(a)	$\frac{1}{2}\times 6^2\times 1.3=...$	M1
	$=23.4\,(\text{m}^2)$	A1
		(2)
(b)	$12.2^2=6^2+10.8^2-2\times 6\times 10.8\cos(ABE)$	M1
	$\cos(ABE)=\frac{6^2+10.8^2-12.2^2}{2\times 6\times 10.8}\left(=\frac{19}{648}\right)$ $ABE=1.54$	A1
		(2)
(c)	$\text{Area } ABE=\frac{1}{2}\times 10.8\times 6\sin(ABE)$	M1
	$\text{Area } BCD=\frac{1}{2}\times 6\cos(\pi-1.3-\text{"1.54"})\times 6\sin(\pi-1.3-\text{"1.54"})$ or e.g. $\text{Area } BCD=\frac{1}{2}\times 6\sin(\pi-1.3-\text{"1.54"})\times\sqrt{6^2-(6\sin(\pi-1.3-\text{"1.54"}))^2}$	M1
	$\text{Total area}=60.9\text{m}^2$	A1
		(3)
		Total 7

Notes:

(a) **Marks for part (a) must be seen in part (a)**

M1: Attempts to evaluate $\frac{1}{2}r^2\theta$ with r = 6 and θ = 1.3 or equivalent e.g. $\frac{\theta}{360}\times\pi\times 6^2$ with $\theta=1.3\times\frac{180}{\pi}$ which may be implied by use of e.g. 74° or 75°

A1: Correct value. Allow equivalent values e.g. $\frac{117}{5}$, $23\frac{2}{5}$.

Condone lack of units but if any are given they should be correct. Correct answer only scores both marks.

(b)

M1: Correct numerical application of the cosine rule for the required angle.

A1: For awrt 1.54

(c) **Allow equivalent work in degrees.**

M1: Correct method for area *ABE* e.g. $\frac{1}{2}\times 10.8\times 6\sin(\text{their } ABE)$

M1: Correct method for area *BCD*

Must be a correct method including the ½

Note that *CD* may be found using the sine rule e.g. $\frac{6}{\sin\frac{\pi}{2}}=\frac{CD}{\sin(\pi-1.3-\text{"1.54"})}\Rightarrow CD=...$

Note that *BC* may be found using the sine rule e.g. $\frac{6}{\sin\frac{\pi}{2}}=\frac{BC}{\sin\left(1.3+\text{"1.54"}-\frac{\pi}{2}\right)}\Rightarrow BC=...$

Having found *CD* or *BC* as above, *BC* or *CD* may be found using Pythagoras.

In such cases the trigonometry must be correct for their values.

Do **not** allow mixing of degrees and radians e.g. $180-1.3-\text{"1.54"}$ for $\pi-1.3-\text{"1.54"}$

A1: Correct total area. Allow awrt 60.9 (Condone lack of units)

<u>Some values for reference:</u>

Angle *DBC* = 0.300…(17.1…°), Angle *BDC* = 1.27…(72.7…°),

Area *ABE* = 32.38…, Area *BCD* = 5.083…, *CD* = 1.77…, *BC* = 5.73…

Question	Scheme	Marks
6(a)	$y-5x=75,\ y=2x^2+x-21$ $\Rightarrow 2x^2+x-21=5x+75$ $\Rightarrow 2x^2-4x-96=0$ or e.g. $x^2-2x-48=0$ or $\Rightarrow y=2\left(\frac{y-75}{5}\right)^2+\frac{y-75}{5}-21$ $\Rightarrow 2y^2-320y+10350=0$ or e.g. $y^2-160y+5175=0$	M1
	$x^2-2x-48=0\Rightarrow(x-8)(x+6)=0\ \Rightarrow x=-6,\ 8$	dM1
	$x=-6\Rightarrow y=45$ or $x=8\Rightarrow y=115$	dM1
	$P(-6,\ 45)$ and $Q(8,\ 115)$	A1
		(4)
(b)	e.g. $y\leqslant 2x^2+x-21,\ y-5x\leqslant 75,\ y\geqslant 0,\ x\leqslant -3.5$ $x\leqslant a$ where $-3.5\leqslant a<3$ (or $a\leqslant x\leqslant b$ where $a\leqslant -15,\ -3.5\leqslant b\leqslant 3$)	M1A1A1
		(3)
		Total 7

Notes:

(a) **<u>Note that solutions relying on calculator technology are not acceptable so a complete method must be shown.</u>**

M1: Equates the line and the curve and rearranges to obtain a 3TQ in *x*
Alternatively eliminates *x* to obtain a 3TQ in *y*

dM1: Solves their 3TQ to obtain at least one value for *x* or *y* by an acceptable method e.g. factorisation, formula or completing the square. They cannot just state the roots without seeing a correct line of intermediate working.

We will condone e.g. $2x^2-4x-96=0\Rightarrow x=\frac{4\pm\sqrt{(-4)^2-4(2)(-96)}}{2\times 2}=-6,\ 8$

But e.g. $2x^2-4x-96=0\Rightarrow x=\frac{4\pm\sqrt{-4^2-4(2)(-96)}}{2\times 2}=-6,\ 8$ scores M0 as it suggests an incorrect formula but allow recovery e.g. $2x^2-4x-96=0\Rightarrow x=\frac{4\pm\sqrt{-4^2-4(2)(-96)}}{2\times 2}=\frac{4\pm\sqrt{784}}{4}=-6,\ 8$ and condone a missing trailing bracket e.g. $x=\frac{4\pm\sqrt{(-4)^2-4(2)(-96}}{2\times 2}=-6,\ 8$

If they factorise do **not** accept $2x^2-4x-96=0\Rightarrow(x+6)(x-8)=0\Rightarrow x=-6,\ 8$

Depends on the previous method mark.

dM1: Uses at least one *x* value to find at least one *y* value (or vice versa) using either of the given equations.
Depends on the <u>FIRST</u> method mark and depends on having solved a 3TQ in *x* or *y*.
May be implied by their values. You may need to check (for both equations).

A1: All correct. Must be correctly paired but ignore any labelling regarding *P* and *Q*, just look for the correct paired values. This may be explicit as a coordinate pair or implied by e.g. *x* = 8, *y* = 115.
Condone coordinates written without brackets e.g. –6, 45
Depends on all previous marks so 1011 is not possible.

Candidates who show <u>no</u> working for solving their 3TQ can score a maximum 1010
Correct answers with no working scores no marks.

(b) **Allow strict or non-strict inequalities and allow a mixture for the first 2 marks.**

M1: Obtains at least 2 of the required inequalities e.g. 2 of:

$$y \leqslant 2x^2 + x - 21,\ y - 5x \leqslant 75,\ y \geqslant 0,\ x \leqslant a \text{ where } -3.5 \leqslant a < 3$$

Allow equivalents e.g. $y < 5x + 75$

Allow e.g. $0 \leqslant y \leqslant 2x^2 + x - 21$ or e.g. $0 < y < 5x + 75$ each of which would count as 2 correct inequalities.

Do **not** allow e.g. $R \leqslant 2x^2 + x - 21$

A1: Any **3** of the 4 required inequalities.

A1: All correct and consistent. For consistency, the inequalities for *y* must be either all strict or all non-strict. The restriction on *x* will depend on their chosen *x* value. For example

$y \geqslant 0,\ y \leqslant 5x + 75,\ y \leqslant 2x^2 + x - 21,\ x < 0$ scores full marks

$y \geqslant 0,\ y \leqslant 5x + 75,\ y \leqslant 2x^2 + x - 21,\ x < -3.5$ scores M1A1A0

If there are extra **incorrect** inequalities then score A0

Question	Scheme	Marks
7(a)(i)	$\mathrm{f}(x) = 2x^3 - kx^2 + 14x + 24 \Rightarrow \left(\mathrm{f}'(x) =\right) 6x^2 - 2kx + 14$	M1A1
(ii)	$\left(\mathrm{f}''(x) =\right) 12x - 2k$	A1ft
		(3)
(b)	$6x^2 - 2kx + 14 = 12x - 2k \Rightarrow 6(5)^2 - 2k(5) + 14 = 12(5) - 2k \Rightarrow k = ...$	M1
	$k = 13$	A1
		(2)
(c)	$k = 13 \Rightarrow 6x^2 - 38x + 40 = 0 \Rightarrow x = ...$	M1
	$x = "\frac{4}{3}" \Rightarrow y = 12\left("\frac{4}{3}"\right) - 2\times 13$ or $y = 6\left("\frac{4}{3}"\right)^2 - 2\times 13\times\left("\frac{4}{3}"\right) + 14$	M1
	$x = \frac{4}{3},\ y = -10$	A1
		(3)
		Total 8

Notes:

In part (a), penalise the inclusion of "+ c" on either derivative only once on the first occurrence.

(a)(i)

M1: For $x^3 \to \ldots x^2$ or $x^2 \to \ldots x$ or $14x \to 14$

A1: Correct simplified first derivative. The $"\mathrm{f}'(x) = "$ is not required.

Do not allow x^1 for x.

Isw can be applied once a correct simplied derivative is seen.

(ii)

A1ft: Correct simplified second derivative. Follow through their first derivative

The $"\mathrm{f}''(x) = "$ is not required.

Isw can be applied once a correct simplied second derivative is seen. **Must follow M1**

(b)

M1: Sets $\mathrm{f}'(x) = \mathrm{f}''(x)$, substitutes $x = 5$ and solves a linear equation in k to find a value for k.

May substitute $x = 5$ into $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$ first, then equate and solve for k.

A1: $k = 13$

(c)

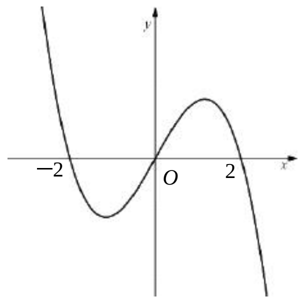
M1: Substitutes their k into the equated $\mathrm{f}'(x) = \mathrm{f}''(x)$ and solves the resulting 3TQ to obtain at least one value for x by any means including a calculator.

M1: Substitutes their x (not 5) and their k into $\mathrm{f}'(x)$ or $\mathrm{f}''(x)$ to find a value for y.

May be implied by their values so you may need to check.

A1: Correct coordinates. May be seen as $x = ..$, $y = \ldots$ or as a coordinate pair but condone missing brackets e.g. $\frac{4}{3}, -10$. Award this mark as soon as the correct values are seen.

Allow a "made up" k in (c).

Question	Scheme	Marks
8(a)		B1B1B1
		(3)
(b)	$x\left(4-x^2\right)=\dfrac{A}{x} \Rightarrow 4x^2-x^4=A$ $\Rightarrow x^4-4x^2+A=0*$	B1*
		(1)
(c)	$A>0$	B1
	$b^2=4ac \Rightarrow 16=4A \Rightarrow A=...$	M1
	$0<A<4$	A1
		(3)
		Total 7

Notes:

(a)

B1: Negative cubic curve in any position. Must have one maximum and one minimum.

B1: Positive or negative cubic curve **passing through** (not touching) the origin.
The origin does not have to be labelled.

B1: Cuts the x-axis at x = 2 and x = –2. May be as shown or as (2, 0) and (−2, 0).

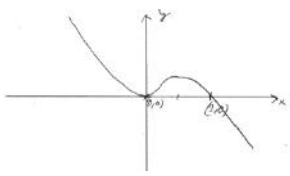
Must cut so extends beyond the x-axis from above or below but **not** a turning point.

Condone (0, 2) and (0, −2) as long as they are in the correct positions.

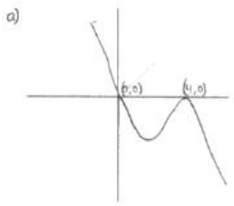
Allow away from the sketch but must be (2, 0) and (−2, 0).

If there is any ambiguity, the sketch takes precedence.

Examples:

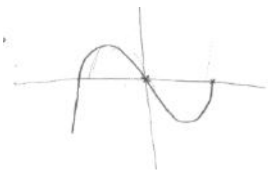


100

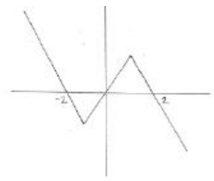


110

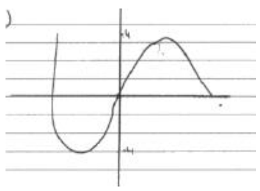
010



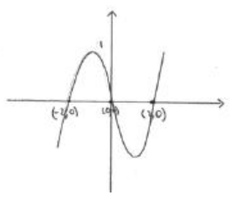
010



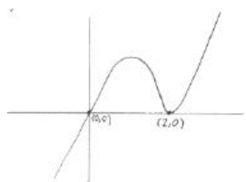
001



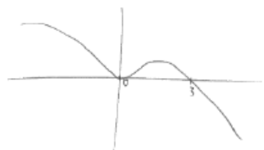
110



011



010



000

(b)

B1*: Equates the curves and shows sufficient working to obtain the printed answer with no errors.

Note that $x\left(4-x^2\right)$ may be expanded first before equating which is fine.

There must be at least one intermediate step between $x\left(4-x^2\right)=\frac{A}{x}$ and the printed answer.

As a minimum accept $x\left(4-x^2\right)=\frac{A}{x}\Rightarrow 4x-x^3=\frac{A}{x}\Rightarrow x^4-4x^2+A=0$

(c)

B1: For $A > 0$ as one boundary. May be seen embedded in their final answer. $x > 0$ is B0.

M1: Attempts $b^2=4ac$ or equivalent e.g. $b^2-4ac=0$ with $a = 1$, $b = -4$ and $c = A$ to obtain a value for A.

Condone use of an inequality rather than "=" to obtain a value or inequality for A e.g. $b^2-4ac>0$ or $b^2-4ac<0$ leading to $A >$ … or $A <$ …

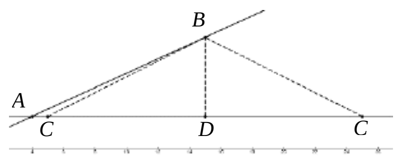
May be implied by e.g. $A = 4$, $A < 4$, $A > 4$ etc.

A1: Correct range for A. Allow equivalents e.g. "$A > 0$ **and** $A < 4$", "(0, 4)"

but not "$A > 0$ **or** $A < 4$", "$A > 0, A < 4$"

Correct answer with no working scores 3/3

Partially correct answer with no working e.g. $0 < A \leqslant 4$ can score B1 M1 (implied) A0

Question	Scheme	Marks
9(a)	$m=\frac{7-2}{15-4}\left(=\frac{5}{11}\right)$	M1
	$y-2=\frac{5}{11}(x-4)$ or $y-7=\frac{5}{11}(x-15)$ or $y=\frac{5}{11}x+c\Rightarrow 2=\frac{5}{11}\times 4+c\Rightarrow c=...\left(\frac{2}{11}\right)$	M1
	$5x-11y+2=0$	A1
		(3)
(a) ALT	$y=mx+c\Rightarrow \begin{matrix}2=4m+c\\7=15m+c\end{matrix}$	M1
	$\begin{matrix}2=4m+c\\7=15m+c\end{matrix}\Rightarrow m=...\left(\frac{5}{11}\right),\quad c=...\left(\frac{2}{11}\right)$	M1
	$5x-11y+2=0$	A1
(b)	$(15-x)^2+(7-2)^2=\left(5\sqrt{5}\right)^2$ or e.g. $\sqrt{(15-x)^2+(7-2)^2}=5\sqrt{5}$ or e.g. $\left(5\sqrt{5}\right)^2=5^2+CD^2$ oe  	M1
	Way 1: $(15-x)^2+25=125\Rightarrow(15-x)^2=100\Rightarrow x=...$ or e.g. Way 2: $125=25+CD^2\Rightarrow CD^2=100\Rightarrow CD=10\Rightarrow x=...$	M1
	$(5,\ 2)$ **or** $(25,\ 2)$	A1
	$(5,\ 2)$ **and** $(25,\ 2)$	A1
		(4)
(c)	Area $=\frac{1}{2}(\text{"5"}-4)\times 5$	M1
	$=\frac{5}{2}$	A1
		(2)
		Total 9

Notes:

(a)

M1: Correct method for the gradient

M1: For a correct straight line method using their gradient and point *A* or point *B* with *x* and *y* correctly placed. Alternatively uses $y = mx + c$ and reaches $c = \ldots$

Note that $\frac{y-2}{7-2}=\frac{x-4}{15-4}$ scores both M marks.

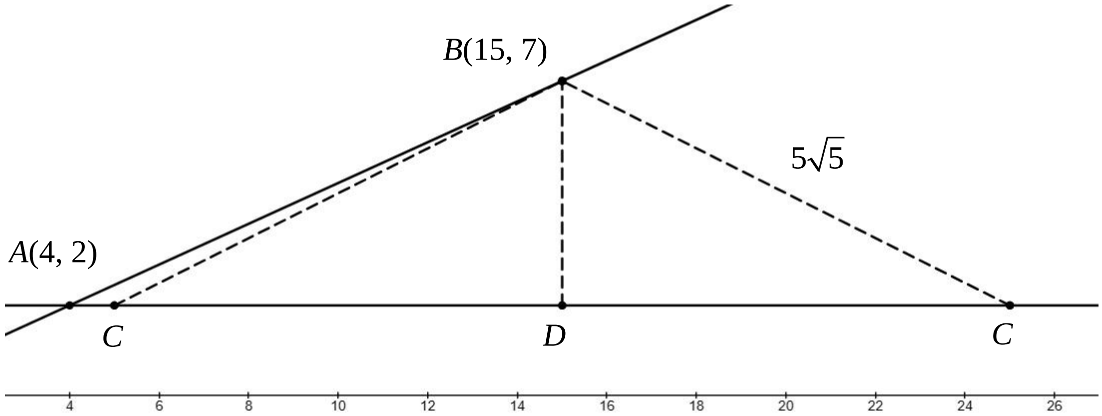
A1: Correct equation in the required form including the "= 0". Allow any integer multiple of this equation.

ALT

M1: Substitutes $x = 4$, $y = 2$ and $x = 15$, $y = 7$ into the general equation $y = mx + c$ to obtain 2 equations in *m* and *c*.

M1: Solves 2 equations of the form $\alpha=m\beta+c$ to find values for *m* and *c*.

A1: Correct equation in the required form including the "= 0". Allow any integer multiple of this equation.



(b)

M1: Correct application of Pythagoras for triangle *BCD* (see diagram above)

The statements shown are sufficient for this mark but condone missing brackets e.g. $5\sqrt{5}^2$ for $\left(5\sqrt{5}\right)^2$ as long as the intention is clear.

M1: A complete, correct and full method to obtain at least **one** *x* coordinate for *C*.

For **Way 1** this mark is for processing an equation of the form: $(15-x)^2+\alpha^2=\left(5\sqrt{5}\right)^2$ where $0<\alpha^2<125$ to obtain at least one value for *x*.

For **Way 2** this mark is for processing an equation of the form: $\left(5\sqrt{5}\right)^2=CD^2+\alpha^2$ where $0<\alpha^2<125$ to obtain at least one value for *CD* and then attempts at least one value of *x* either 15 – *CD* or 15 + *CD*. Condone the use of *x* (or any other letter) for *CD*.

The work for this mark must be correct so e.g. $\sqrt{(15-x)^2+(7-2)^2}=5\sqrt{5}\Rightarrow(15-x)+(7-2)=5\sqrt{5}$

or e.g. $\left(5\sqrt{5}\right)^2=CD^2+\alpha^2\Rightarrow 5\sqrt{5}=CD+\alpha$ scores M0. If a 3TQ is being solved, the usual rules apply.

A1: At least one correct position for *C*.

May be seen as a coordinate pair with or without brackets or as e.g. $x = 5$, $y = 2$

A1: Both correct pairs of coordinates.

May be seen as coordinate pairs with or without brackets or as e.g. $x = 5$, $y = 2$

Note that some candidates think the equation for l_2 is $y = 4$ rather than $y = 2$
This scores a maximum of 1 mark in part (b) and will look like this:

$$(15-x)^2+(7-4)^2=\left(5\sqrt{5}\right)^2\Rightarrow(15-x)^2+(7-4)^2=125-9=116$$

$$(15-x)=\pm2\sqrt{29}\Rightarrow x=15\pm2\sqrt{29}$$

or

$$\left(5\sqrt{5}\right)^2=3^2+CD^2\Rightarrow CD^2=116\Rightarrow CD=2\sqrt{29}$$

$$\Rightarrow x=15\pm2\sqrt{29}$$

This scores M0M1A0A0

(c)

M1: Applies a correct triangle area for one of their points of the form (*k*, 2), $k \neq 15$. May be implied.

This mark is for e.g. $\frac{1}{2}(k-4)\times5$ where *k* is one of their calculated *x* coordinates from part (b)

or for $\frac{1}{2}(11\pm CD)\times5$ where *CD* is their calculated value from part (b)

May see shoelace method: $\frac{1}{2}\begin{vmatrix}4 & 15 & "5" & 4\\ 2 & 7 & 2 & 2\end{vmatrix}=\frac{1}{2}\left|4\times7+15\times2+"5"\times2-2\times4-7\times"5"-2\times15\right|$

Allow other correct methods for the area e.g. using trigonometry.

Their *C* must be of the form (*k*, 2) and a correct formula used or implied.

A1: Correct minimum area. Must be 2.5 **not** awrt 2.5 (e.g. if they use trigonometry and get an inexact answer)

Question	Scheme	Marks
10(a)	$f'(4)=6(4)-\frac{7\times 14}{4}=-\frac{1}{2}$	B1
	$m_T=-\frac{1}{2}\Rightarrow m_N=\frac{-1}{-1/2}$	M1
	$y-12=2(x-4)$ or $y=mx+c\Rightarrow y=2x+c\Rightarrow 12=2(4)+c\Rightarrow c=...$	M1 (A1 on ePen)
	$y=2x+4$	A1
		(4)
(b)	$...\frac{(2x-1)(3x+2)}{2\sqrt{x}}=...\frac{6x^2+x-2}{2\sqrt{x}}=...3x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}-x^{-\frac{1}{2}}$	M1
	$f(x)=\frac{6x^2}{2}-\frac{6}{5}x^{\frac{5}{2}}-\frac{1}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}(+c)$ or e.g. $f(x)=\frac{6x^2}{2}-\frac{3}{5/2}x^{\frac{5}{2}}-\frac{1/2}{3/2}x^{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{1/2}(+c)$ or e.g. $f(x)=\frac{6x^2}{2}-\left(\frac{3}{5/2}x^{\frac{5}{2}}+\frac{1/2}{3/2}x^{\frac{3}{2}}-\frac{x^{\frac{1}{2}}}{1/2}\right)(+c)$	M1A1A1
	$12=\frac{6(4)^2}{2}-\frac{6}{5}(4)^{\frac{5}{2}}-\frac{1}{3}(4)^{\frac{3}{2}}+2(4)^{\frac{1}{2}}+c\Rightarrow c=...$	M1
	$(f(x)=)3x^2-\frac{6}{5}x^{\frac{5}{2}}-\frac{1}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+\frac{16}{15}$	A1
		(6)
		Total 10

Notes:

(a)

B1: Correct gradient at P (may be implied)

M1: Attempts to use the perpendicular gradient rule to find the normal gradient.

Look for e.g. $m_N=\frac{-1}{m_T}$ or e.g. $m_T\times m_N=-1\Rightarrow -\frac{1}{2}m_N=-1\Rightarrow m_N=...$

May be implied by their normal gradient.

M1(A1 on Epen): Attempts the equation of the normal using a "changed" gradient with x = 4 and y = 12 correctly placed.

Alternatively uses $y = mx + c$ with a "changed" gradient with x = 4 and y = 12 to find a value for c.

A1: Correct equation in the required form

(b) **Allow work for part (b) seen in part (a) to score in (b) provided it is seen or used in (b)**

M1: Expands the numerator of the fraction and attempts to split.

Score for one correct index achieved from correct work e.g. $\frac{..x^2}{\sqrt{x}} \to ...x^{\frac{3}{2}}$ or $\frac{...x}{\sqrt{x}} \to ...x^{\frac{1}{2}}$ or $\frac{...}{\sqrt{x}} \to ...x^{-\frac{1}{2}}$

M1: Attempts to integrate a fractional power. E.g. $...x^{\frac{3}{2}} \to ...x^{\frac{5}{2}}$ or $x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$ or $x^{-\frac{1}{2}} \to ...x^{\frac{1}{2}}$ etc.

Do not allow this mark for an attempt to integrate the $\sqrt{x}$ in the denominator.

A1: Any 2 correct terms simplified or unsimplified.

A1: All correct simplified or unsimplified. The "+ *c*" is not required here.

M1: Uses $x = 4$ and $y = 12$ following an attempt to increase at least one power *x* by 1 to find a value for their *c*.

Their *c* may not be fully evaluated but must be a numerical expression.

A1: Correct simplified form. The $"\text{f}(x) = "$ is not required just look for the correct expression.

Accept equivalent simplified forms and allow $(\text{f}(x) =) 3x^2 - \left(\frac{6}{5}x^{\frac{5}{2}} + \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right) + \frac{16}{15}$

Apply isw if applicable unless they multiply all terms by e.g. 15

Question	Scheme	Marks
11(a)	$x = \frac{5\pi}{2}$ **or** $y = 12$	B1
	$x = \frac{5\pi}{2}$ **and** $y = 12$	B1
		(2)
(b)	$x = \frac{3\pi}{2}$ **or** $y = -21$	B1
	$x = \frac{3\pi}{2}$ **and** $y = -21$	B1
		(2)
(c)(i)	$(A =) -12$	B1
(ii)	$(B =) \frac{5\pi}{4}$	B1
		(2)
		Total 6

Notes:

Look for answers written in the body of the question or on the sketch.

In all cases, answers written in the body of the script take precedence over answers written on the diagram.

Angles and numbers must be written as <u>single</u> terms e.g. not $2\pi + \frac{\pi}{2}$ for $\frac{5\pi}{2}$

Penalise the use of degrees <u>once</u> the first time it occurs

e.g. (a) (450°, 12) (b) (270°, −21) (c) $A = -12$, $B = 225°$ scores (a) B1B0 (b) B1B1 (c) B1B1

The degrees symbol is not required

If both coordinates in (a) and (b) are correct but the wrong way round score as B1B0 each time

e.g. (a) $\left(12, \frac{5\pi}{2}\right)$ (b) $\left(-21, \frac{3\pi}{2}\right)$ scores (a) B1B0 (b) B1B0

If the coordinates in (a) or (b) are the wrong way round and only one is correct score as B0B0 each time

If the coordinates are the wrong way round <u>and</u> correct in degrees in (a) and (b)

e.g. (a) (12, 450°) (b) (−21, 270°) award (a) B0B0 (b) B1B0

The degrees symbol is not required

(a)

B1: One coordinate correct.

May be seen as $x = \ldots$ or $y \ldots$ or embedded in a coordinate pair e.g. $\left(\frac{5\pi}{2}, 12\right)$ or in a vector $\begin{pmatrix} \frac{5\pi}{2} \\ 12 \end{pmatrix}$

B1: Both coordinates correct.

May be seen as $x = \ldots$ and $y \ldots$ or as a coordinate pair e.g. $\left(\frac{5\pi}{2}, 12\right)$ or as a vector $\begin{pmatrix} \frac{5\pi}{2} \\ 12 \end{pmatrix}$

(b)

B1: One coordinate correct.

May be seen as $x = \ldots$ or $y \ldots$ or embedded in a coordinate pair e.g. $\left(\frac{3\pi}{2}, -21\right)$ or in a vector $\begin{pmatrix} \frac{3\pi}{2} \\ -21 \end{pmatrix}$

B1: Both coordinates correct.

May be seen as $x = \ldots$ and $y \ldots$ or as a coordinate pair e.g. $\left(\frac{3\pi}{2}, -21\right)$ or as a vector $\begin{pmatrix} \frac{3\pi}{2} \\ -21 \end{pmatrix}$

(c)(i)

B1: Correct value for A. The "$A =$" is not required but it must be clear it is the answer to (i) or is the value of A.

(ii)

B1: Correct value for B. The "$B =$" is not required but it must be clear it is the answer to (ii) or is the value of B.

If candidates write the values as a coordinate pair e.g. $\left(\frac{5\pi}{4}, -12\right)$ or $\left(-12, \frac{5\pi}{4}\right)$ so it is not clear which value is which, score B0B0